MECE 480 Final Solutions 2017-2018 spring

Q1. $G(z) = \dfrac{4(z+2)}{z(z+0.5)}$ $\quad C(z) = \dfrac{P(z)}{L(z)} = \dfrac{P(z)}{L'(z)\hat{L}(z)}$

To have dead-beat control for reference step input

$\hat{L}(z) = z-1$

should be second order

$$C(z) = \frac{p_0 + p_1 z + p_2 z^2}{(l_0' + l_1' z)(z-1)}$$

5 point

$$z^4 = 1 + G(z)C(z)$$

$$z^4 = 1 + \frac{4(z+2)}{z(z+0.5)} \frac{(p_0 + p_1 z + p_2 z^2)}{(l_0' + l_1' z)(z-1)}$$

10 point

$$z^4 = (z^2 + 0.5z)\left(l_1' z^2 + (l_0' - l_1')z - l_0'\right) + (4z+8)(p_0 + p_1 z + p_2 z^2)$$

$$z^4 = [l_1'] z^4 + (0.5 l_0' - 0.5 l_1' - l_0') z^2 + [0.5 l_1' + l_0' - l_1'] z^3 - l_0' 0.5 z$$
$$+ 4 p_2 z^3 + (4p_1 + 8p_2) z^2 + (8p_1 + 4p_0) z + 8p_0$$

$$z^4 = l_1' z^4 + (l_0' - 0.5 l_1' + 4p_2) z^3 + (-0.5 l_0' - 0.5 l_1' + 4p_1 + 8p_2) z^2 + (-0.5 l_0' + 8p_1 + 4p_0) z + 8p_0$$

1 point 1 point 15 point

$l_1' = 1$ $\quad p_0 = 0$ $\quad l_0' - 0.5 l_1 + 4p_2 = 0$ $\quad (-0.5 l_0' - 0.5 l_1' + 4p_1 + 8p_2) = 0$

$(-0.5 l_0' + 8p_1 + 4p_0) = 0$

$$\boxed{l_0^1 + 4p_2 = 0.5}$$
3 point

$$\boxed{-0.5l_0^1 + 8p_1 = 0}$$
3 point

$$\boxed{-0.5l_0^1 + 4p_1 + 8p_2 = 0.5}$$
3 point

$$-0.5l_0^1 + 8p_1 = 0$$
$$8p_1 = 0.5l_0^1$$
$$\boxed{l_0^1 = 16p_1}$$

$$-0.5l_0^1 + 4p_1 + 8p_2 = 0.5$$
$$2l_0^1 \quad + 8p_2 = 1$$
$$-$$
$$-2.5l_0^1 + 4p_1 = -0.5$$
$$-2.5(16p_1) + 4p_1 = -0.5 \Rightarrow -40p_1 + 4p_1 = -0.5$$
$$-36p_1 = -0.5$$
$$p_1 = \frac{1}{72}$$
2 point

$$l_0^1 = \frac{16}{72}$$
2 point

$$l_0^1 + 4p_2 = 0.5$$
$$\frac{16}{72} + 4p_2 = \frac{1}{2} \longrightarrow \frac{16}{72} + 4p_2 = \frac{36}{72}$$
$$4p_2 = \frac{20}{72}$$
$$p_2 = \frac{5}{72}$$
3 point

$$C(z) = \frac{0 + \frac{1}{72}z + \frac{5}{72}z^2}{\left(\frac{16}{72} + z\right)(z-1)} = \frac{\frac{5}{72}z^2 + \frac{1}{72}z + 0}{z^2 - \frac{56}{72}z - \frac{16}{72}}$$
2 point

Q2

$$x_{k+1} = \underbrace{\begin{bmatrix} 0 & 0.4 \\ 0.1 & 0 \end{bmatrix}}_{A_d} x_k + \underbrace{\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}}_{b_d} u_k \qquad y_k = \underbrace{[a_3 \; a_4]}_{c_d^T} x_k$$

(a) Eigenvalues

$$[zI - A_d] = \begin{bmatrix} z & -0.4 \\ -0.1 & z \end{bmatrix} \qquad \det[(zI - A_d)] = z^2 - 0.04 = (z-0.2)(z+0.2)$$

$$z_1 = 0.2 \qquad z_2 = -0.2$$

10 puan

(b) Controllability $Q = \begin{bmatrix} a_1 & 0.4a_2 \\ a_2 & 0.1a_1 \end{bmatrix} = [b_d \;\; A_d b_d]$

10 puan

$$\det Q = 0.1a_1^2 - 0.4a_2^2 = 0.1[a_1^2 - 4a_2^2] = 0.1[a_1 + 2a_2][a_1 - 2a_2]$$

if $a_1 = -2a_2$ or $a_1 = 2a_2$ this system is uncontrollable

$a_1 = 2 \quad a_2 = 1$ (so not completely controllable)

(c) Transfer function $a_1 = 2 \quad a_2 = 1 \quad a_3 = 1 \quad a_4 = 1$

$$T(z) = c_d^T (zI - A_d)^{-1} b_d = [a_3 \;\; a_4] \begin{bmatrix} z & -0.4 \\ -0.1 & z \end{bmatrix}^{-1} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

10 puan

$$= \frac{[a_3 \;\; a_4] \begin{bmatrix} z & 0.4 \\ 0.1 & z \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}}{z^2 - 0.04}$$

$$= \frac{[a_3 \;\; a_4] \begin{pmatrix} a_1 z + 0.4a_2 \\ a_1 0.1 + a_2 z \end{pmatrix}}{z^2 - 0.04}$$

$$= \frac{[1 \;\; 1] \begin{bmatrix} 2z + 0.4 \\ \cancel{2z} + 0.2 \end{bmatrix}}{z^2 - 0.04} = \frac{2z + 0.4 + z + 0.2}{(z - \cancel{0.02})(z + 0.2)} = \frac{3(z+0.2)}{(z-0.2)(z+0.2)}$$

(c) $T(z) = \dfrac{3}{z-0.2}$

(d) Use state feedback to replace the poles to $z_1 = 0$, $z_2 = 0$

Is it possible

$$A_d + b_d k_d^T = \begin{bmatrix} 0 & 0.4 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} [k_1 \ k_2]$$

$$\tilde{A}_d = A_d + b_d k_d^T = \begin{bmatrix} 0 & 0.4 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 2k_1 & 2k_2 \\ k_1 & k_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2k_1 & 0.4+2k_2 \\ 0.1+k_1 & k_2 \end{bmatrix}$$

10 puan

$$zI - \tilde{A}_d = \begin{bmatrix} z-2k_1 & -(0.4+2k_2) \\ -(0.1+k_1) & z-k_2 \end{bmatrix}$$

$$\det(zI - \tilde{A}_d) = (z-2k_1)(z-k_2) - (-(0.4+2k_2))(-(0.1+k_1))$$

$$= z^2 - (2k_1+k_2)z + 2k_1k_2 - \left[2k_1k_2 + (2k_2)0.1 + k_1 0.4 + 0.02\right]$$

$$= z^2 - (2k_1+k_2)z + 2k_1k_2 - 2k_1k_2 - 0.2k_2 - 0.4k_1 - 0.04$$

$$= z^2 - (2k_1+k_2)z - (0.2k_2 + 0.4k_1 + 0.04)$$

$$= z^2 - (2k_1+k_2)z - \frac{1}{5}\left[2k_1+k_2+0.2\right]$$

$$= (z+0.2)(z-0.2-2k_1-k_2)$$

(B-)

$$\det(zI - \tilde{A}_d) = (z+0.2)(z-0.2-2k_1-k_2) \overset{?}{=} z^2$$

not possible

(e) Now use state feedback to replac the poles to $z_1 = 0$ $z_2 = -0.2$

$$\det(zI - \tilde{A}_d) = (z+0.2)(z-0.2-2k_1-k_2) = z(z+0.2)$$

$$-0.2 - 2k_1 - k_2 = 0$$

$$0.2 + 2k_1 + k_2 = 0$$

$$2k_1 + k_2 = -0.2$$

$k_2 = 0$

$k_1 = -0.1$

10 poeng