2018-2019 spring

MECE 480 Final Solutions

(Q1) $G(z) = \dfrac{2(z+1)}{z(z-0.5)}$

$$C(z) = \frac{P(z)}{L(z)} = \frac{P(z)}{L'(z)\hat{L}(z)}$$

To have dead-beat control for reference step input

$\hat{L}(z) = z - 1$

$$C(z) = \frac{p_0 + p_1 z + p_2 z^2}{(l_0' + l_1' z)(z-1)}$$ 10 puan

$$z^4 = 1 + G(z)\,C(z)$$

$$z^4 = 1 + \frac{2(z+1)}{z(z-0.5)} \frac{(p_0 + p_1 z + p_2 z^2)}{(l_0' + l_1' z)(z-1)}$$

10 puan

$$z^4 = z(z-0.5)(l_0' + l_1' z)(z-1) + (2z+2)(p_0 + p_1 z + p_2 z^2)$$

$$z^4 = (z^2 - 0.5z)\left(l_1' z^2 + (l_0' - l_1')z - l_0'\right) + 2p_2 z^3 + (2p_1 + 2p_2)z^2 + (2p_0 + 2p_1)z + 2p_0$$

$$z^4 = l_1' z^4 + (l_0' - l_1' - 0.5 l_1')z^3 + (-l_0' - 0.5 l_0' + 0.5 l_1')z^2 + 0.5 l_0' z$$
$$+ 2p_2 z^3 + (2p_1 + 2p_2)z^2 + (2p_0 + 2p_1)z + 2p_0$$

10 puan

$$z^4 = l_1' z^4 + (l_0' - l_1' - 0.5 l_1' + 2p_2) z^3 + (-l_0' - 0.5 l_0' + 0.5 l_1' + 2p_1 + 2p_2) z^2 + (0.5 l_0' + 2p_0 + 2p_1) z + 2p_0$$

10 равн

5 равн

$$l_1' = 1, \quad p_0 = 0$$

$$l_0' - 1.5 l_1' + 2p_2 = 0 \qquad -1.5 l_0' + 0.5 l_1' + 2p_1 + 2p_2 = 0$$

$$0.5 l_0' + 2p_0 + 2p_1 = 0$$

5 равн

$$l_0' - 1.5 + 2p_2 = 0 \qquad -1.5 l_0' + 0.5 + 2p_1 + 2p_2 = 0$$

$$\boxed{l_0' + 2p_2 = 1.5} \qquad \boxed{-1.5 l_0' + 2p_1 + 2p_2 = -0.5}$$

$$0.5 l_0' + 2p_1 = 0$$

$$\boxed{l_0' = -4p_1}$$

$$\boxed{-4p_1 + 2p_2 = 1.5} \qquad \boxed{6p_1 + 2p_1 + 2p_2 = -0.5}$$

$$\boxed{8p_1 + 2p_2 = -0.5}$$

$$-12p_1 = 2 \qquad p_1 = -\frac{1}{6}$$

$$\rho_1 = \frac{-1}{6} \Rightarrow -4\left(-\frac{1}{6}\right) + 2\rho_2 = 1.5 \Rightarrow \frac{2}{3} + 2\rho_2 = \frac{3}{2}$$

$$2\rho_2 = \frac{9-4}{6} = \frac{5}{6} \qquad \rho_2 = \frac{5}{12}$$

$$\ell_0' = -4\rho_1 = -4\left(-\frac{1}{6}\right) = \frac{2}{3}$$

$$C(z) = \frac{\rho_0 + \rho_1 z + \rho_2 z^2}{(\ell_0' + \ell_1' z)(z-1)} = \frac{0 + \left(-\frac{1}{6}\right)z + \frac{5}{12} z^2}{\left(\frac{2}{3} + 1z\right)(z-1)}$$

$$= \frac{\frac{5}{12} z^2 - \frac{1}{6} z}{\left(z + \frac{2}{3}\right)(z-1)} = \frac{z\left(\frac{5}{12} z - \frac{1}{6}\right)}{\left(z + \frac{2}{3}\right)(z-1)}$$

W pan

Q1

$l_0' - 1.5 l_1' + 2p_2 = 0$

$0.5 l_0' + 2p_0 - 2p_1 = 0$

in case $G(z) = \frac{2(z-1)}{z(z-0.5)}$

$-1.5 l_0' + 0.5 l_1' + 2p_1 - 2p_2 = 0$

$-2p_0 = 0$

$l_1' = 1$

$l_0' - 1.5 + 2p_2 = 0$

$0.5 l_0' - 2p_1 = 0$

$-1.5 l_0' + 0.5 + 2p_1 - 2p_2 = 0$

$l_0' + 2p_2 = 1.5$ $\rightarrow$ $4p_1 + 2p_2 = 1.5$

$0.5 l_0' = 2p_1$

$l_0' = 4p_1$

$-1.5(4p_1) + 0.5 + 2p_1 - 2p_2 = 0$

$-6p_1 + 2p_1 - 2p_2 = -0.5$

$-4p_1 - 2p_2 = -0.5$

$4p_1 + 2p_2 = 1.5$

$-4p_1 - 2p_2 = -0.5$

$\rightarrow$ no solution

Q2 $x_{k+1} = \begin{bmatrix} 0 & -0.9 \\ 0.1 & 0 \end{bmatrix} x_k + \begin{bmatrix} a \\ b \end{bmatrix} u_k \qquad y_k = \begin{bmatrix} 1 & 1 \end{bmatrix} x_k$

(a) $zI - A_d = \begin{bmatrix} z-0 & +0.9 \\ -0.1 & z-0 \end{bmatrix} = z^2 + 0.09 = 0 \qquad z_{1,2} = \mp j0.3$

10 points

(b) $Q_c = \begin{bmatrix} b_d & b_d A_d \end{bmatrix} = \begin{bmatrix} 0 & -0.9b \\ b & 0.1a \end{bmatrix} \qquad \det(Q_c) = 0.1a^2 + 0.9b^2$

10 points

if $a=0$ and $b=0$ system is not completely controllable
otherwise system is completely controllable

(c) $G(z) = c_d^T \left[zI - A_d \right]^{-1} b_d$

$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} z & 0.9 \\ -0.1 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix} \dfrac{\begin{bmatrix} z & -0.9 \\ 0.1 & z \end{bmatrix}}{z^2 + 0.09} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

12 points

$= \dfrac{\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} z - 0.9 \\ z + 0.1 \end{bmatrix}}{z^2 + 0.09} = \dfrac{z - 0.9 + z + 0.1}{z^2 + 0.09} = \dfrac{2z - 0.8}{z^2 + 0.09}$

(d) $A_d + b_d k_d^T = \begin{bmatrix} 0 & -0.9 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \tilde{A}_d$

$= \begin{bmatrix} 0 & -0.9 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} k_1 & k_2 \\ k_1 & k_2 \end{bmatrix}$

$= \begin{bmatrix} k_1 & k_2 - 0.9 \\ k_1 + 0.1 & k_2 \end{bmatrix}$

8 points

$$(zI - \tilde{A}_d) = \begin{bmatrix} z-k_1 & 0.9-k_2 \\ -(0.1+k_1) & z-k_2 \end{bmatrix}$$

$$\det(zI - \tilde{A}_d) = (z-0)(z-0) = z^2$$

(new pole location)

$$\det(zI - \tilde{A}_d) = (z-k_1)(z-k_2) + (0.9-k_2)(0.1+k_1)$$

$$= z^2 - (k_1+k_2)z + k_1k_2 - k_1k_2 + (0.9k_1 - 0.1k_2) + 0.09$$

$$z^2 = z^2 - (k_1+k_2)z + (0.9k_1 - 0.1k_2) + 0.09$$

$$-(k_1+k_2) = 0 \qquad 0.9k_1 - 0.1k_2 + 0.09 = 0$$

(8 points)

$$k_1 = -k_2 \longrightarrow -0.9k_2 - 0.1k_2 + 0.09 = 0$$

$$0.09 = k_2 \qquad k_1 = -0.09$$

<u>sağlama</u>

$$\hat{A}_d = \begin{bmatrix} 0 & -0.9 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} -0.09 & 0.09 \end{bmatrix}$$

$$\hat{A}_d = \begin{bmatrix} 0 & -0.9 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} -0.09 & 0.09 \\ -0.09 & 0.09 \end{bmatrix}$$

$$\tilde{A}_d = \begin{bmatrix} -0.09 & -0.81 \\ 0.01 & 0.09 \end{bmatrix} \qquad zI - \hat{A}_d = \begin{bmatrix} z+0.09 & 0.81 \\ -0.01 & z-0.09 \end{bmatrix}$$

$$z^2 - 0.0081 + 0.0081 = 0 \qquad z^2 = 0 \qquad z_1 = 0 \quad z_2 = 0$$