

Homework II

MECE 480 ~~HW~~
2015-2016 spring

Q1

$$x_{k+1} = \underbrace{\begin{bmatrix} -1 & -0.5 \\ -1 & -0.5 \end{bmatrix}}_{A_d} x_k + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{b_d} u_k \quad y_k = \underbrace{[1 \ 0]}_{c_d^T} x_k$$

(a) $T(z) = c_d^T [zI - A_d]^{-1} b_d$

$$= [1 \ 0] \begin{bmatrix} z+1 & 0.5 \\ 1 & z+0.5 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [1 \ 0] \frac{\begin{bmatrix} z+0.5 & -0.5 \\ -1 & z+1 \end{bmatrix}}{(z+1)(z+0.5) - (-1)(-0.5)} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{[1 \ 0] \begin{bmatrix} z \\ z \end{bmatrix}}{z^2 + 1.5z + 0.5 - 0.5} = \frac{z}{z^2 + 1.5z} = \frac{1}{z + 1.5}$$

(b) eigenvalues of $(zI - A_d) = 0 \Rightarrow (z+1)(z+0.5) - (-1)(-0.5) = 0$

$$= z^2 + 1.5z = 0 \begin{cases} \rightarrow z_1 = 0 \\ \rightarrow z_2 = -1.5 \end{cases}$$

$$\textcircled{c} \quad \tilde{A}_d = A_d + b_d k_d^T \quad \left(\text{to put all eigenvalues to } z=0 \begin{matrix} \nearrow \lambda_1=0 \\ \searrow \lambda_2=0 \end{matrix} \right)$$

$$= \begin{bmatrix} -1 & -0.5 \\ -1 & -0.5 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [k_1 \ k_2]$$

$$\tilde{A}_d = \begin{bmatrix} k_1-1 & k_2-0.5 \\ k_1-1 & k_2-0.5 \end{bmatrix}$$

$$zI - \tilde{A}_d = \begin{bmatrix} z+1-k_1 & 0.5k_2 \\ 1-k_1 & z+0.5k_2 \end{bmatrix}$$

$$\det(zI - \tilde{A}_d) = (z+(1-k_1))(z+(0.5k_2)) - (1-k_1)(0.5k_2)$$

$$= z^2 + (1+0.5k_1-k_2)z$$

$$\neq z^2 + [1.5-k_1-k_2]z = z[z-(1.5k_1-k_2)]$$

$$\neq z^2 + [1.5-k_1-k_2]z \neq z^2$$

$$1.5-k_1-k_2=0 \quad 1.5=k_1+k_2$$