

$$\text{Ex: } G(s) = \frac{1}{s^3(s+1)} \Rightarrow G(z) = \frac{z^{-1}}{z} Z \left(\mathcal{L}^{-1} \left(\frac{1}{s} G(s) \right) \right) \Big|_{t=kT}$$

$$\frac{1}{s} G(s) = \frac{1}{s^3(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{s+1}$$

$$C = \frac{d^2}{ds^2} \frac{1}{s+1} \Big|_{s=0} = 1$$

$$B = \frac{d}{ds} \frac{1}{s(s+1)} \Big|_{s=0} = \frac{0 \times (s+1) - 1 \times 1}{(s+1)^2} \Big|_{s=0} = -1$$

$$A = \frac{d^2}{ds^2} \frac{1}{s+1} \Big|_{s=0} = \frac{d}{ds} \frac{1}{(s+1)^2} \Big|_{s=0} = -2$$

$$\frac{1}{s^3(s+1)} = \frac{As^2 + Bs + C}{s^3} + \frac{D}{s+1}$$

$$1 = [As^2 + Bs + C](s+1) + Ds^3$$

$$1 = (A+D)s^3 + (A+B)s^2 + (C+B)s + C$$

$$C = 1$$

$$B = -1$$

$$A = 1$$

$$D = -1$$

$$\frac{1}{s} G(s) = \frac{1}{s^3(s+1)} = \frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^3} - \frac{1}{s+1}$$

$\downarrow \mathcal{L}^{-1}$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} G(s) \right\} = g(t) = \left(g(t) + \frac{t^2}{2} g(t) - e^{-t} g(t) \right)$$

$$\left. \mathcal{L} \left\{ \frac{1}{s} G(s) \right\} \right|_{t=kT} = G_k - (kT) \dot{G}_k + \frac{(kT)^2}{2} \ddot{G}_k - e^{-kT} \ddot{G}_k //$$

$$\begin{matrix} (z) \\ \downarrow \end{matrix} \quad z \left\{ \left. \mathcal{L} \left\{ \frac{1}{s} (G(s)) \right\} \right|_{t=kT} \right\} = \frac{z}{z-1} - \frac{Tz}{(z-1)^2} + \frac{1}{2}$$